Research Project (Progetto di Ricerca)

Borel-definable refinements of classical algebraic invariants in topology have been recently introduced by Bergfalk, Lupini, and Panagiotopoulos in the context of Borel-definable homological algebra. This framework builds on initial intuitions of Moore and Beilinson–Bernstein–Deligne of replacing abelian groups as target category of algebraic invariants with a canonical "abelian envelope" of the category of abelian Polish groups. Such a category is called the *left heart*, and was later constructed and characterized by Schneiders for arbitrary quasi-abelian categories. The application of methods from descriptive set theory gave new impetus to this approach, leading to an explicit description of the left heart of abelian Polish groups in the work of Lupini. In this description, the objects are *groups with a Polish cover*, namely groups explicitly presented as the quotient of an abelian Polish group (the Polish cover) by a Polish subgroup (whose topology makes the inclusion continuous, but generally not a topological embedding). The morphisms are group homomorphisms that are *Borel-definable*, in the sense that admit a lift to a Borel function between the Polish covers. Analogous descriptions hold for the left heart of a number of important categories of algebraic structures endowed with a topology, including locally compact abelian Polish groups and separable Banach spaces. In this project, the successful applicant will apply this framework to the study of homological invariant of locally compact groups, including Moore's Borel cohomology, continuous cohomology, and bounded cohomology.

Ext of locally compact groups

Since the early days of the development of homological algebra, researchers tried to extend the techniques and results obtained in the discrete setting to the topological case. This lead to the work of Mackey, Moskowitz, Fulp, Griffith and others on extensions of locally compact groups and the algebrica invariant Ext parametrizing such extensions. This invariant was set on a firm homological footing by Hoffmann and Spitzweck, who recognized it as a the cohomological derived functor of Hom on the left heat of the category of locally compact groups. Building on this insight, Bergfalk, Lupini, Moraschini, and Sarti applied the concrete description of such a left heart outlined above to completely characterize injective and projective objects in the left heart of locally compact groups as well as a number of important subcategories.

Building on this work, this project will characterize the complexity of Ext for locally comapact groups, extending the work of Lupini in the discrete case. These result will be applied to measure the complexity of classification problems admitting extensions of locally compact groups as invarinats.

Cohomology of locally compact groups

A cohomology theory for locally compact groups was introduced by Moore in the 1970s and later studied by Austin, Moore, and DuPre, among others. The goal of this project is reconcile Moore's concrete approach and the abstract homological algebra approach, recognizing group cohomology as a derived functor on the left heart of locally compact groups, in analogy with the work of Hoffmann and Spitzwek for Ext. This will allow one to regard cohomology of locally compact groups as a functor to the left heart of Polish groups.

As an application, we will obtain an Universal Coefficient Theorem relating cohomology and homology for locally compact groups in terms of Ext. In turn, this will make it possible to apply the complexity computations for Ext to obtain analogous results for cohomology.

The same techniques will be applied tot he study of continuous cohomology, which will be shown to be a derived functor in the more general context of relative homological algebra for exact categories.

Bounded cohomology

Bounded cohomology is of countable discrete groups was extended by Monod to the context of locally compact groups. This invariant can be seen as a functor to the left heart of the category of Banach spaces. The goal of this project is to obtain a concrete description for such a category analogous to the one for Polish groups, and to use this description to define a definable refinement of bounded cohomology for countable and locally compact groups. The next task will be to obtain this invariant as a derived functor, and to compute its complexity. As an application, we will establish a Universal Coefficient Theorem relating bounded cohomology with ℓ^1-cohomology, in analogy with the classical Universal Coefficient Theorem of Eilenberg and MacLane relating homology and cohomology of spaces.

Work-plan (Piano di Attività)

The successful applicant will engage in the following activities:

1. Review of the literature on Borel-definable refinements of classical algebraic invariants.

2. Review of the literature on algebraic and homological invariants of locally compact groups.

3. Work on the relevant items of the research project as listed above, including collaborations with Lupini and other staff members at UNIBO as well as collaborators from other institutions. This phase will involve research trips to visit collaborators.

4. Present the result of the research at research seminars at the University of Bologna and other Universities, and at relevant conferences as invited or contributed speaker.

5. Write research papers on the results of the research, to be submitted to prestigious peer-reviewed journals.

6. Write a survey paper on applications of Borel-definable methods to algebraic invariants of locally compact groups, to be submitted to the Bulletin of Symbolic Logic or another scientific journal that publishes surveys and expository articles.